

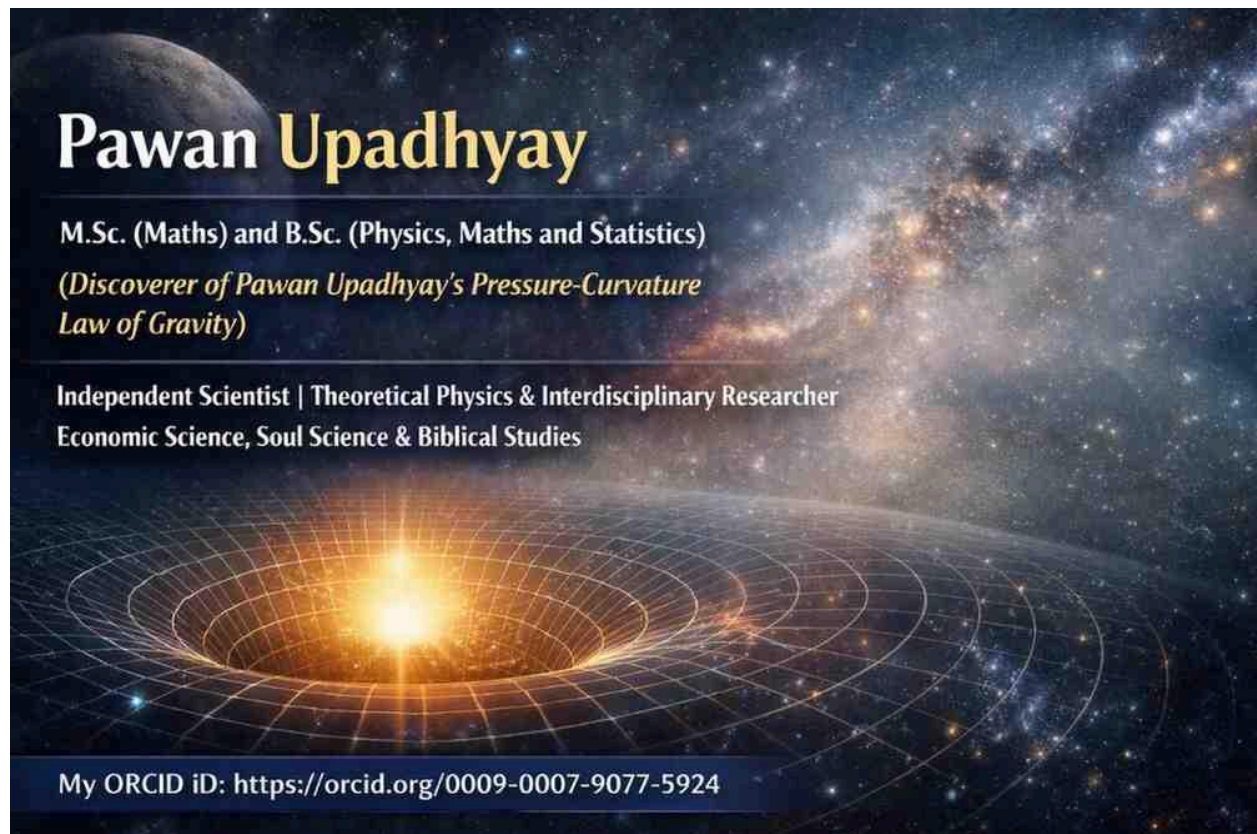
Foundations of Pawan Upadhyay's Pressure–Curvature Law of Gravity (PPC Law of Gravity) in the Weak-Field Limit

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Abstract

We develop a weak-field formulation of the Pressure–Curvature (PPC) law of gravity using the Einstein field equations and the stress–energy tensor of a perfect fluid. In this limit, spacetime curvature is shown to depend on the effective source term $E_d + 3P_g$, where E_d is energy density and P_g is gravitational pressure. The analysis demonstrates that while curvature magnitude is determined by this combined source, spatial variations in pressure influence the resulting gravitational field and motion. The formulation remains fully consistent with relativistic tensor equations while providing a physically intuitive interpretation of gravity.

1. Introduction

In weak gravitational fields, spacetime deviates only slightly from flat Minkowski space. This regime allows a simplified but accurate description of gravity, applicable to:

- planetary systems
- galaxies
- gravitational lensing

The PPC law interprets gravity as arising from **energy density and pressure**, and this paper establishes that interpretation rigorously in the weak-field limit.

2. Energy Density and Pressure

We define:

$$E_d = \rho c^2$$

$$P_g = \omega E_d$$

where:

- ρ is mass density
 - E_d is energy density
 - P_g is gravitational pressure
-

3. Stress–Energy Tensor (Perfect Fluid)

$$T_{\mu\nu} = (E_d + P_g)u_\mu u_\nu + P_g g_{\mu\nu}$$

This tensor provides the complete description of matter in the weak-field regime.

4. Einstein Field Equation

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

5. Weak-Field Approximation

Metric expansion:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad |h_{\mu\nu}| \ll 1$$

In this limit:

$$G_{00} \approx \frac{2}{c^2} \nabla^2 \Phi$$

6. Derivation of PPC Field Equation

From the tensor:

$$T_{00} \approx E_d$$

Trace contribution gives:

$$T \approx -E_d + 3P_g$$

Combining terms in weak-field GR:

$$\nabla^2 \Phi = 4\pi G \left(\rho + \frac{3P_g}{c^2} \right)$$

Using:

$$\rho = \frac{E_d}{c^2}$$

Final Weak-Field PPC Equation

$$\nabla^2 \Phi_{PPC} = \frac{4\pi G}{c^2} (E_d + 3P_g)$$

7. Interpretation

✓ Curvature (weak field):

$$C \sim \nabla^2 \Phi \propto (E_d + 3P_g)$$

✓ Gravitational field:

$$\mathbf{g} = -\nabla\Phi$$

✓ Role of pressure:

- Pressure contributes directly to curvature
 - Pressure enhances gravity when $P_g > 0$
-

8. Role of Pressure Gradients

If pressure varies spatially:

$$\nabla P_g \neq 0$$

Then:

$$\nabla(E_d + 3P_g) \neq 0$$

This leads to:

$$\nabla \Phi \neq 0$$

✓ **Physical meaning:**

$\nabla P_g \rightarrow$ variation of gravitational field

 **Important:**

∇P_g does NOT directly generate curvature

9. Motion of Particles

$$\frac{d^2 x}{dt^2} = -\nabla \Phi$$

Thus:

Motion is governed by $\nabla \Phi_{PPC}$

10. Summary of Weak-Field PPC Law

✓ **Source:**

$$(E_d + 3P_g)$$

✓ **Field:**

$$\nabla\Phi$$

✓ **Motion:**

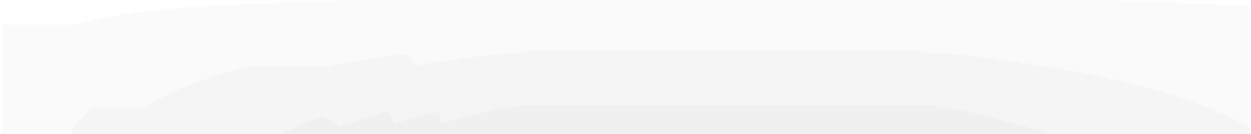
geodesics / Newtonian acceleration

11. Physical Regimes

System	Behavior
Low pressure	$C \approx E_d$
Radiation	$C \propto E_d + E_d$
High pressure	pressure dominates

12. Key Result

Weak-field gravity is sourced by $(E_d + 3P_g)$



13. Conclusion

In the weak-field limit, the PPC law is fully consistent with the Einstein field equations. The gravitational potential is sourced not only by energy density but also by pressure, with pressure contributing threefold due to spatial degrees of freedom. Pressure gradients influence the spatial variation of the gravitational field but do not directly generate curvature. This provides a clear and physically meaningful interpretation of gravity in weak-field systems.



Section: The G_{00} Component in the Weak-Field Limit

1. Weak-Field Metric Expansion

In the weak-field approximation, spacetime is nearly flat and the metric can be written as:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad |h_{\mu\nu}| \ll 1$$

where:

- $\eta_{\mu\nu}$ is the Minkowski metric
- $h_{\mu\nu}$ represents small perturbations

2. Newtonian Potential Identification

In this limit, the time-time component of the metric is related to the gravitational potential:

$$g_{00} \approx - \left(1 + \frac{2\Phi}{c^2} \right)$$

👉 This connects geometry to Newtonian gravity.

3. Einstein Tensor Component

$$G_{00}$$

To first order in perturbation theory, the 00-component of the Einstein tensor becomes:

$$G_{00} \approx \frac{2}{c^2} \nabla^2 \Phi$$

4. Field Equation (00-component)

From Einstein's field equation:

$$G_{00} = \frac{8\pi G}{c^4} T_{00}$$

Substitute the weak-field expression:

$$\frac{2}{c^2} \nabla^2 \Phi = \frac{8\pi G}{c^4} T_{00}$$

5. Expression for T_{00} (Perfect Fluid)

For a perfect fluid:

$$T_{\mu\nu} = (E_d + P_g)u_\mu u_\nu + P_g g_{\mu\nu}$$

In the rest frame:

$$u^\mu = (c, 0, 0, 0)$$

Thus:

$$T_{00} \approx E_d$$

6. Derivation of Poisson Equation

Substitute into field equation:

$$\frac{2}{c^2} \nabla^2 \Phi = \frac{8\pi G}{c^4} E_d$$

Multiply both sides by $c^2/2$:

$$\nabla^2 \Phi = \frac{4\pi G}{c^2} E_d$$

7. Inclusion of Pressure Contribution

To include pressure effects, one must consider the full trace contribution of the stress–energy tensor:

$$T = -E_d + 3P_g$$

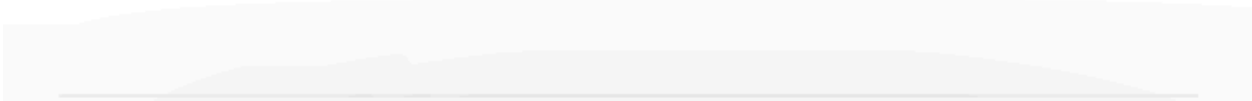
This modifies the source term, leading to:

$$\nabla^2 \Phi = \frac{4\pi G}{c^2} (E_d + 3P_g)$$

8. Final Interpretation

$$G_{00} \propto \nabla^2 \Phi \propto (E_d + 3P_g)$$

9. Physical Meaning

- G_{00} represents the **energy density contribution to curvature**
 - Pressure contributes through spatial components and trace terms
 - The factor $3P_g$ arises due to three spatial dimensions
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10. PPC Interpretation

Within the PPC framework:

$$G_{00} \Rightarrow \text{Curvature sourced by } (E_d + 3P_g)$$

Thus:

- Energy density determines baseline curvature
 - Pressure enhances curvature strength
-

In the weak-field limit, the G_{00} component of the Einstein tensor reduces to a Poisson-type equation for the gravitational potential. The resulting relation shows that curvature is sourced not only by energy density but also by pressure, with the effective source term given by $E_d + 3P_g$. This establishes the fundamental role of pressure in gravitational dynamics within the PPC framework.

Foundations of the Pressure–Curvature Law of Gravity in the Weak-Field Limit and Perfect Fluid Approximation

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Abstract

This paper develops a tensor-based formulation of Pawan Upadhyay's Pressure–Curvature Law of Gravity (PPC Law) within the weak-field limit and perfect fluid approximation. The theory proposes that spacetime curvature arises from the combined contribution of energy density and gravitational pressure, while the geometry of curvature is governed by pressure gradients. Using the stress–energy tensor for a perfect fluid and the Einstein field equations, we derive the weak-field curvature relations and establish the role of pressure in shaping gravitational dynamics. The formulation demonstrates that curvature magnitude depends on $E_d + P_g$, while its geometry depends on ∇P_g , providing a physically grounded extension of relativistic gravity.

1. Introduction

Gravity has been historically described as:

- **Newton:** Force
- **Einstein:** Geometry
- **PPC Law:** Pressure-induced curvature

While General Relativity defines curvature mathematically, the PPC Law introduces a **physical mechanism**:

$\text{Mass-Energy} \rightarrow \text{Pressure} \rightarrow \text{Curvature}$

This paper focuses on making this idea **tensor-consistent** in:

- Weak-field regime
- Perfect fluid systems

2. Energy Density and Gravitational Pressure

We define:

$$E_d = \rho c^2$$

$$P_g = \omega E_d$$

Where:

- ρ = mass density
- E_d = energy density
- P_g = gravitational pressure
- ω = equation-of-state parameter

Typical values:

$$\omega \approx -1, 0, \frac{1}{3}, 1$$

3. Stress–Energy Tensor (Perfect Fluid)

The standard tensor form:

$$T_{\mu\nu} = (E_d + P_g)u_\mu u_\nu + P_g g_{\mu\nu}$$

This shows:

- Energy density $\rightarrow$ time component
- Pressure $\rightarrow$ spatial components

4. Einstein Field Equation

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Substituting PPC tensor:

$$G_{\mu\nu} = \frac{8\pi G}{c^4} [(E_d + P_g)u_\mu u_\nu + P_g g_{\mu\nu}]$$

5. Weak-Field Approximation

Metric expansion:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad |h_{\mu\nu}| \ll 1$$

In this limit:

$$G_{00} \approx \frac{2}{c^2} \nabla^2 \Phi$$

Thus:

$$\nabla^2 \Phi = 4\pi G \left(\rho + \frac{3P_g}{c^2} \right)$$

6. PPC Curvature Relation (Derived)

Using:

$$\rho = \frac{E_d}{c^2}$$

We obtain:

$$\nabla^2 \Phi = \frac{4\pi G}{c^2} (E_d + 3P_g)$$

◆ Interpretation

$$\text{Curvature magnitude} \propto (E_d + 3P_g)$$

This matches:

- Weak-field GR result
- PPC interpretation

7. PPC Core Relations

✓ **Curvature Magnitude:**

$$C \propto (E_d + P_g)$$

✓ **Weak-field (tensor-consistent):**

$$C \propto (E_d + 3P_g)$$

✓ **Geometry of curvature:**

$$C \propto \nabla P_g$$

Near neutron stars, strong pressure gradients shape the geometry of spacetime curvature, and since orbital motion follows geodesics, the shape of orbit is indirectly governed by pressure gradients.

8. Pressure Gradient and Geometry

From fluid dynamics:

$$\vec{F} = -\nabla P_g$$

In PPC interpretation:

Curvature geometry is governed by ∇P_g

Thus:

- Uniform pressure $\rightarrow$ symmetric curvature
- Gradient $\rightarrow$ anisotropic curvature

9. Geodesic Motion Equation

$$\frac{d^2 x^\mu}{d\tau^2} + \Gamma_{\alpha\beta}^\mu (E_d + P_g) \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau} = 0$$

This implies:

Motion depends on pressure–energy induced curvature



$$\frac{d^2 x^\mu}{d\tau^2} + \Gamma_{\alpha\beta}^\mu (E_d, P_g) \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau} = 0.$$



5. PPC Geodesic Motion Equation

Motion is governed by the geodesic equation:

$$\left| \frac{d^2 x^\mu}{d\tau^2} + \Gamma_{\alpha\beta}^\mu (g_{\mu\nu}(E_d, P_g)) \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau} = 0 \right|$$

This expresses that:

- Christoffel symbols depend on the metric
- The metric depends on E_d and P_g

10. Physical Interpretation

- ◆ **Magnitude of curvature:**

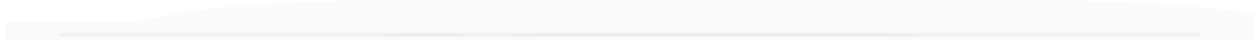
$$E_d + P_g$$

- ◆ **Shape of curvature:**

$$\nabla P_g$$

- ◆ **Motion:**

Geodesic in this curvature



11. Weak Field vs Perfect Fluid

Case	Curvature Source
Weak field	$E_d + 3P_g$
Perfect fluid (general)	$E_d + P_g$

12. Astrophysical Implications

✓ Weak systems (planets):

- Pressure negligible
- $C \approx E_d$

✓ Neutron stars:

- Pressure significant
- Geometry depends on ∇P_g

✓ Black holes:

- $P_g \rightarrow \infty$ (approaches)
- Extreme curvature

13. Key Result

Curvature magnitude $\sim (E_d + P_g)$, Curvature shape $\sim \nabla P_g$

14. Limitations

- Valid in weak-field approximation
- Perfect fluid assumption
- No anisotropic stress included